Thermal effects in laser pumped Kerr-lens modelocked Ti:sapphire lasers

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Abstract

Numerical beam propagation simulations are used to demonstrate that the distributed thermal lensing, produced by the absorption of the pump laser in the gain medium, profoundly affects the operation of hard-apertured, Kerr-lens modelocked Ti:sapphire lasers. The pump-induced thermal lensing is shown to shift and distort the resonator stability regions (even allowing the regions to overlap) and severely perturb the modelocking mechanism in one of the stability regions.

1. Introduction

In recent years, Kerr-lens modelocked (KLM) femtosecond Ti:sapphire lasers [1] have become some of the most important ultrafast spectroscopic tools. During their continued development, which has led to novel modelocking techniques [2–6] and lasers generating < 10 fs optical pulses [7,8], a number of theoretical investigations have led to a deeper understanding of the operation of these laser systems. In particular, the effects of cavity alignment [9–16], gain guiding [16–19], diffraction [20], and dispersive pulse broadening [21] on KLM solid-state lasers have been considered. In comparison, the effect that the distributed thermal lens, produced by the absorption of the pump laser beam in the gain medium, has on the operation of femtosecond solid-state lasers has received relatively little attention. Two analyses [22,23] have approximated the thermal lens by truncating the pump-induced temperature distribution after the quadratic term and treated the thermal lens as a single intracavity element in the paraxial approximation. In neither case was the effect of the thermal lens on the propagation of the pump beam itself addressed, which will clearly influence the exact distribution in the gain medium of both the pump-induced thermal lens and the gain profile experienced by the intracavity laser beam. On the other hand, pump-induced thermal lensing effects have received appreciable attention in end-pumped CW solid-state lasers [24–26], such as diode laser pumped Nd:YAG, where they influence the efficiency of fundamental spatial mode operation. In addition, it should be noted that it has been suggested that thermal lensing aids [27,28] and stabilizes [29] the KLM mechanism in solid-state lasers.

In this paper, we present a rigorous treatment of the propagation of both the pump and intracavity laser beams through the gain medium using a weighted least squares fit to describe the thermal and nonlinear Kerr lensing [30,31] in the paraxial self-consistent Gaussian beam propagation approximation. This treatment of the gain medium is included in a resonator stability analysis of a canonical linear cavity to demonstrate that the pump-induced thermal lens has a major impact on the operational characteristics of hard-apertured KLM Ti:sapphire lasers producing pulse durations of 50–100 fs. In particular, for typical pumping conditions and gain media greater than 1 cm in length, we show that (i) the thermal lens shifts and distorts the resonator stability regions to such an extent that overlap between the two stability regions is possible, and (ii) modelocked operation in one of the stability regions is strongly perturbed by the thermal lens. The numerical analysis also demonstrates that the propagation of both the
pump and intracavity laser beams through the gain medium is affected by the pump-induced thermal lens, indicating that the thermal lens will alter the nature of gain guiding and saturation effects in the KLM mechanism [16–19]. However, in order to study in isolation the role which thermal lensing plays in the KLM operation of Ti:sapphire resonators, these important gain related effects, and those due to aperture diffraction [20], are omitted in our analysis.

In our theoretical model, which is described in Section 2, we divide the Ti:sapphire gain medium into discrete elements and use a radially symmetric graded-index (GRIN) lens approximation [22,30–33] to treat both the thermal and Kerr lensing effects in each element. The radial approximation used in the numerical beam propagation simulation is not expected to affect the validity of our results since a comparison of Refs. [10] and [11] with Ref. [12], which treated the Kerr lensing case in the absence of thermal effects, illustrates that the inclusion of a Brewster-cut gain medium and astigmatism compensation does not significantly alter the cavity conditions required for KLM operation. In Section 3 to allow comparison to be made with the thermal case, our Gaussian beam resonator analysis is initially applied to the case when only the Kerr lensing is present. Thereafter, the pump beam parameters and focusing conditions, which are critical to an investigation of thermal effects in laser-pumped solid-state lasers, are discussed in Section 4. The treatment of the pump-induced distributed thermal lens (the thermo-optic effect) is presented in Section 5, and its effect on KLM operation is analyzed in Section 6. Finally, in Section 7, we discuss the influence of thermal expansion and stress induced refractive index changes [26,34] which have been omitted in our analysis.

2. The laser resonator

The radial Gaussian beam propagation analysis presented in this paper treats the canonical hard-apertured KLM laser resonator shown schematically in Fig. 1. The gain section of the resonator consists of a 20 mm-long Ti:sapphire crystal placed between two 10 cm radius of curvature mirrors (represented as lenses $f_1$ and $f_2$). For consistency with the notation of other authors [9–13], the distance between the gain section mirrors is denoted by $z$, and $x$ represents the position of the gain medium with respect to the mirror $f_1$ nearest the output coupler $M_1$. The resonator is chosen to have unequal arm lengths ($d_1 = 80$ cm and $d_2 = 100$ cm) [15] and has a typical overall length of $\sim 1.9$ m for a repetition rate of $\sim 80$ MHz. The slit to control KLM operation is assumed to be in the shorter arm of the resonator next to the output coupler. Therefore, to assess the potential for hard-apertured modelocked operation, the model evaluates the change in the spot size of the intracavity radiation at mirror $M_1$ between short-pulse modelocked (ML) operation and normal, continuous-wave (CW) cavity oscillation.

The propagation of both the intracavity laser beam and the pump laser beam are treated using the ABCD matrix formalism in the paraxial Gaussian beam approximation. The Ti:sapphire laser rod is divided into 1000 elements to allow the Kerr and pump-induced thermal lensing effects to be treated in a thin GRIN lens approximation [22,32,33]. The radial refractive index changes $\Delta n$ associated with both lensing effects are approximated (using a weighted least squares fit) to the required parabolic dependence for each individual GRIN lens element along the gain medium. It is important to note that it is highly problematic to treat the thermal lens in the single ABCD matrix approximation [30] which has been successfully employed to describe the effect Kerr-lensing in solid-state laser resonators [9–13]. This is because, unlike the laser irradiance dependent Kerr-lensing, the strongest thermal lensing may not occur at the pump beam’s focal point since the pump suffers absorption in the Ti:sapphire gain medium. Furthermore, the positional strength of the Kerr-lensing in the gain medium is affected by the thermal lensing since the latter influences the propagation of the intracavity laser mode through the gain medium. It is for these reasons that we chose to use a finite element GRIN lens analysis.

The numerical simulation of the intracavity oscillation begins by selecting a plane-wave (i.e. focal) spot size at
the high reflector $M_2$ and propagating the Gaussian mode through the resonator to the output coupler $M_1$ for a particular value of the rod position $x$ in the absence of Kerr lensing. The two possible values for the gain section mirror separation $z$ are then found for which the laser mode is also a plane-wave at the output coupler $M_1$. This procedure establishes the allowed CW oscillation modes for a given spot size at the high reflector $M_2$ in the two stability regions characteristic of this type of laser resonator [9–16]. For each fixed pair of values $(x, z)$ and in the presence of the Kerr lens, the procedure is now repeated iteratively for different spot sizes at the high reflector $M_2$ to find the oscillation conditions for ML operation. In this manner, the characteristics of the oscillating Gaussian modes for both CW and ML operation are found so that the spot sizes at the output coupler can be compared to assess the potential for hard-apertured KLM operation. By repeating the process for different initial spot sizes at the high reflector, both cavity stability regions are completely investigated.

We emphasize that the above unidirectional cavity stability analysis is sufficient under the approximation used in this paper that the nonlinear Kerr lensing can be characterized in terms of a laser power level which is a fraction of the critical power for self focusing $P_c$ [9–15]. This means that variations in the intracavity pulse power due to cavity losses (at the output coupler, etc.) and dispersive pulse broadening [21] are neglected. In addition, other effects such as gain guiding [16–19] which can change the spot size of the oscillating intracavity mode are neglected. Under these approximations, we have verified the validity of considering only propagation in one direction by demonstrating the reciprocity of the cavity [9–11,14]; i.e. that propagation of the intracavity mode back through the resonator, with both the Kerr and static pump-induced thermal lenses present in the gain medium, reproduces the initial Gaussian beam at the high reflector $M_2$. This test of reciprocity also demonstrates that a mesh size of 1000 elements in the gain medium, which produces a propagation error of less than 2%, is more than sufficient.

3. Nonlinear refraction

The nonlinear optical lensing of the intracavity radiation due to the Kerr effect is included in the radial GRIN lens approximation using the weighted least squares parabolic fit to the laser beam profile described in Refs. [30] and [31]. Specifically, a parabolic nonlinear refractive index variation $\Delta n(r)$ of the form

$$\Delta n(r) = \frac{2n_2 P}{\pi w_L^2} \left( B_K - A_K \frac{2r^2}{w_L^2} \right)$$

is used, where $w_L$ is the spot size of the intracavity laser beam in the gain medium, $P$ is the laser power, $A_K$ and $B_K$ are the parabolic fitting parameters for the Kerr lens, and the coefficient of nonlinear refraction $n_2 = 3 \times 10^{-20}$ m$^2$/W for sapphire at the laser wavelength of 800 nm [22,31,35]. In this formulation, with $A_K = 0.5$ and $B_K = 0.75$ [30,31], our GRIN lens model predicts a critical power $P_c \approx 2.2$ MW for the self focusing of a circular plane-wave Gaussian laser beam – a value which is consistent with previous results [14,36,37].

In accordance with previous work [9–13,19], we use the derivative (evaluated for $P \to 0$) of the intracavity laser beam spot size at the mode locking aperture $w_a$ (i.e. at the slit next to the output coupler $M_1$ for the cavity depicted in Fig. 1) with respect to the normalized laser power to determine the ability of the resonator to sustain hard-apertured KLM operation. For laser powers $P < 0.3 P_c$ in which case the variation in $w_a$ with $P$ is approximately linear [8,35], this “Kerr-lens sensitivity” parameter $\delta$ can be written as [9–13,38]

$$\delta = \left( \frac{1}{w_a} \frac{d w_a}{d (P/P_c)} \right)_{P \to 0} \approx \frac{P_c}{P} \left( \frac{w_K}{w_{CW}} - 1 \right),$$

where $w_{CW}$ and $w_K$ are the laser spot sizes at the output coupler $M_1$ (the slit position) for CW oscillation and ML operation respectively. In all the numerical resonator simulations presented in this paper, we use a power of $0.25P_c$ which corresponds to an intracavity pulse energy of $\sim 50$ nJ for a 100 fs pulse. For this laser power the self-consistent Gaussian beam propagation approximation used in our paraxial code is valid [39].

To establish the validity of our numerical simulation and allow the influence of the pump-induced thermal lens to be accurately identified, we first investigated the operational characteristics of the resonator without any thermal lensing, i.e. in the absence of the pump. The results are shown in Fig. 2 in the form of a contour plot of the parameter $\delta$ as a function of $x$ and $z$ for both stability regions of the resonator. For clarity only the contours for $\delta > 0$ (white) are shown in steps of 0.4 to $\delta < -1.6$ (black).
\(\delta < 0\) are shown since hard-aperture KLM requires \(w_K < w_{\text{CW}}\) at the slit position. The results are in close agreement with those obtained by other authors under similar conditions using the single ABCD matrix approximation for the Kerr lensing \([9-13]\). In particular, they show that KLM operation is most strongly favored for \(x = 40\) mm in stability region I nearest the gap between the two stability regions (z \(> 102.5\) mm). In stability region II, mode-locked operation is again favored nearest the stability gap (z \(< 103.5\) mm) at \(x = 52.5\) mm, but the discrimination between CW and ML oscillation in hard-aperture KLM operation is not as pronounced.

4. Pump beam parameters

Any quantitative numerical investigation of the effects produced by the pump beam in laser-pumped solid-state KLM oscillators, such as thermal lensing \([22,23,29]\) and gain saturation \([16-19]\), requires the use of a realistic set of pump beam parameters. Aside from the pump power \(P_o\), which is usually determined by the available pump laser, the most important pump beam parameters are the choice of focusing conditions into the gain medium. For low threshold laser oscillation, a good overlap must exist between the pump and intracavity laser beam profiles in the gain medium. In solid-state laser resonators where the length of the gain medium \(L\) is shorter than the Rayleigh range (i.e. \(L < n\pi w_0^2/\lambda_o\)) associated with the focused intracavity laser mode between the gain section mirrors (e.g. oscillators producing \(\sim 10\) fs pulses \([7,8]\)), good pump/laser beam overlap will exist if the pump beam has roughly the same spot size as the lasing mode in the gain medium. However, this condition is not optimum when \(L > n\pi w_0^2/\lambda_o\), as is the case for the resonator analyzed in this paper (Fig. 1), since the dissimilar pump and laser wavelengths ensure that the two Gaussian beams diverge at different rates. A good compromise is to choose pump focusing conditions under which the pump and laser beam foci coincide in the gain medium, and which give the pump beam the same divergence (i.e. Rayleigh range) as the intracavity laser beam in the gain medium. For an \(Ar^+\) ion pumped Ti:sapphire laser, the latter condition implies that the pump's focal spot size should be \(\sim 30\%\) smaller than that of the intracavity laser mode \([40,41]\). Unfortunately, for the fixed pump focusing conditions usually employed in laser-pumped solid-state lasers, this set of circumstances cannot be maintained for arbitrary stable configurations of the resonator depicted in Fig. 1 since the focal spot size of the intracavity laser mode in the gain medium is a function of the gain section mirror separation \(z\).

To avoid introducing additional variables in the numerical simulation, we chose the following fixed set of pump beam parameters typical of \(Ar^+\) ion pumped Ti:sapphire lasers: a pump power \(P_o = 6\) W (assumed to be at a single wavelength of 500 nm) focused to a spot size of \(12\) \(\mu\)m at a point coincident with the focal point of the intracavity laser mode between the gain section mirrors. For a pump absorption coefficient \(\alpha = 1\) cm\(^{-1}\) used in our analysis, which is typical of Ti:sapphire lasers with 2 cm-long gain media, we found that these pump focusing conditions gave the best pump/laser overlap in the gain medium throughout most of the two resonator stability regions. However, we emphasize that these conditions are not necessarily optimal for the preferred regions of KLM operation near the inner edges of the two resonator stability regions (see Fig. 2) where the focal laser mode size in the gain medium is generally less than \(10\) \(\mu\)m. Indeed, a comparison of the integrated pump/laser overlap over the gain medium under CW and ML operation revealed that the Kerr lensing causes a deterioration in the pump/laser overlap near the inner stability edge of region II, which would result in a shift of the optimum point for KLM oscillation away from the inner stability edge towards the center of this stability region. On the other hand, we found that the pump/laser overlap between CW and ML oscillation did not change appreciably near the inner edge of stability region I. Naturally, these observations are expected to be a function of the choice of pump focusing conditions (e.g. the relative positions of the pump and laser foci), which implies that the pump focusing conditions can be tailored for optimum KLM operation. This point is often overlooked in the design of laser-pumped KLM solid-state lasers.

5. The thermo-optic effect

In laser-pumped solid-state lasers like Ti:sapphire, Cr:LiSAF, Cr:forsterite, etc., the origin of all thermal effects is the Stokes shift between the pump and emission wavelengths. For example, even if the quantum efficiency is near unity \([34,42]\), approximately \(38\%\) of the pump laser power must be dissipated as heat in an \(Ar^+\)-pumped (\(\lambda_{\text{pump}} = 500\) nm) Ti:sapphire laser emitting at 800 nm. The heat is usually removed by external cooling of the gain medium. The resulting temperature gradient is responsible for the thermal effects in solid-state gain media \([24-26,34,43]\). The dominant thermal effect in nearly all solid-state laser crystals is the thermo-optic effect, i.e. the refractive index change with temperature. In Ti:sapphire, this results in a positive pump-induced lensing mechanism since the variation of the refractive index with temperature \(dn/dT = 12.6 \times 10^{-6} \text{ K}^{-1}\) is positive \([34]\).

To evaluate the strength of the pump-induced thermal lensing in the Ti:sapphire gain medium, the temperature distribution generated by the absorption of the pump radiation must be evaluated. For a Gaussian heat source term of the form \(\exp(-2r^2/w_0^2)\) describing the pump beam of spot size \(w_0\), the temperature distribution \(T(r)\) can be
obtained from a series solution to the steady-state radial thermal diffusion equation [22,23,25]:
\[
k \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + Q \exp\left(-2r^2/w_p^2\right) = 0
\]
with
\[
Q = \frac{2\alpha SP_0 \exp\left(-\alpha \xi\right)}{\pi w_p^2}.
\]
where \(Q\) is the absorbed power per unit volume for each element in our GRIN lens approximation, \(P_0\) is the incident pump power, \(\xi\) is the coordinate describing the propagation direction of the pump beam in the gain medium, and \(k = 42 \text{ Wm}^{-1}\text{K}^{-1}\) is the thermal conductivity for sapphire [22,23,34]. The parameter \(S = 0.38\) represents the fraction of the absorbed pump power that is converted to heat. The full series solution to Eq. (3) can be expressed as [25]
\[
T(r) - T_0 = \frac{Qw_p^2}{8k} \sum_{m=1}^{\infty} \left(-1\right)^m \left(\frac{2r^2}{w_p^2}\right)^m,
\]
where \(T_0\) is the peak on axis temperature at \(r = 0\). In obtaining this series solution, we have neglected thermal diffusion in the direction parallel to the propagation of the pump beam. This is a very good approximation because \(\alpha^{-1} \gg w_p\) for all realistic values of \(w_p\) and \(\alpha\) in femtosecond Ti:sapphire lasers. The form of \(\Delta T(r) = T(r) - T_0\) is shown in Fig. 3 as a function of \(r/w_p\) together with the Gaussian spatial profile of the pump heat source. For illustrative purposes, the temperature change \(\Delta T(r)\) has been evaluated for a pump spot size \(w_p = 12 \mu m\), \(\alpha = 1 \text{ cm}^{-1}\), and an \(\text{Ar}^+\) pump power of 2.2 W — conditions that might typically occur at the center of the Ti:sapphire gain medium used in our simulations.

In Fig. 3 we also show two possible parabolic temperature distributions for use in the GRIN lens approximation of the pump-induced thermal lensing. The first (dotted line), which has been used by other authors to describe the pump-induced thermal lensing [22,23], is produced by truncating Eq. (4) after the \(m = 1\) term. The second (dashed line) is the parabolic approximation to \(\Delta T(r)\) obtained using a weighted least squares fit equivalent to the one employed by Magni et al. [30] and Bridges et al. [31] for the Kerr lensing. In other words, the dashed curve is described by the function
\[
\Delta T(r) = \frac{Qw_p^2}{4k} \left( B_T - A_T \frac{2r^2}{w_p^2} \right),
\]
two beams differ throughout the gain medium. In the weighted least squares approximation, this limitation is overcome by evaluating the parabolic fit using the irradiance profile of the intracavity laser beam (i.e. $\exp(-2r^2/w_p^2)$) as the weighting function. Fig. 4 shows the result of this procedure through a plot of the parabolic fitting parameters $A_T$ and $B_T$ for arbitrary ratios of the laser and pump beam spot sizes $w_L/w_p$. For the fitting parameter $A_T$, which is proportional to the strength of the thermal lens, the results display the expected trends. When $w_L < w_p$, the first term of the series solution for $T(r)$ is retrieved since $A_T = 1$. And when $w_L = w_p$, which is also the case describing the thermal lensing experienced by the pump beam, the strength of the thermal lens is approximately halved since $A_T = 0.5$.

In the GRIN lens numerical simulation presented in this paper, the pump-induced thermal lens will be described by Eq. (5) with the parameter $A_T$ obtained from the more accurate weighted least squares parabolic fit. For arbitrary pump/laser spot size ratios, the variation of the fitting parameter $A_T$ with $w_L/w_p$ is well represented by a Gaussian function of the form

$$A_T(w_L/w_p) = 0.334 + 0.681 \times \exp\left[-1.21\left(w_L/w_p + 0.0624\right)^2\right] \quad (6a)$$

$$= \frac{1}{2} + \frac{1}{2} \exp\left[-8(w_L/w_p)^2\right], \quad (6b)$$

which is illustrated by the solid line in Fig. 4. The parabolic variation in the refractive index change $\Delta n(r)$ due to thermal effects can now be evaluated independently for both the pump and intracavity laser beams using Eqs. (5) and (6a) and the refractive index change with temperature $dn/dT$ for sapphire [34]. Thus, we are now in a position to investigate the effect of pump-induced thermal lensing on the KLM mechanism in femtosecond solid-state laser resonators. The only required modification to the numerical model described in Section 2 is that the pump beam be propagated through the gain medium first to establish the form of the thermal duct encountered by the intracavity laser beam in both CW and ML operation. In doing so, we rigorously include the influence of the thermal lens on the pump beam itself by iteratively propagating the pump beam through the 1000 GRIN lens elements comprising the Ti:sapphire crystal in the numerical analysis. For each GRIN lens element, the absorbed pump power per unit volume $Q$ is found using Eq. (3), and then Eq. (5) is employed with $A_T(w_L/w_p = 1) = 0.522$ (Eq. (6a)) and the value of $dn/dT$ for sapphire [34] to evaluate the radial parabolic refractive index change affecting the propagation of the pump beam. A representative example of the influence of the thermal lens on the transmission of the incident 6 W Ar$^+$ pump beam through the 20 mm Ti:sapphire gain medium in the absence of bulk lensing effects. The reduction in the pump spot size, which is caused by the positive thermal lensing, produces an enhancement of $\sim 15\%$ in the maximum peak on-axis pump irradiance. In other words, the pump-induced thermal lensing, by virtue of its effect on the propagation of the pump beam, will modify the gain and spatial gain profile experienced by the intracavity laser beam and, hence, the exact role that gain saturation and guiding effects [16-19] play in the KLM mechanism.

The pump-induced thermal lensing should also affect the spatial overlap between the pump and laser beams in the Ti:sapphire crystal since, from the above analysis, dissimilar pump and laser spot sizes produce different thermal lensing strengths as the two beams propagate through the gain medium. However, we consistently found that the best overlap over the majority of both cavity stability regions again occurred when the pump and laser wavefronts incident on the gain medium had coincident foci in the absence of thermal and Kerr lensing effects. We therefore employed the same constant pump focusing conditions used in Section 4 with the pump beam propagating in the same direction as the intracavity laser beam during the numerical simulation, i.e. from the high reflector ($M_2$) towards the output coupler ($M_1$) in Fig. 1.
6. Cavity analysis with thermal lensing

In the presence of the pump-induced thermal lens in the Ti:sapphire gain medium, the laser resonator shown in Fig. 1 is expected to exhibit different cavity stability criteria [22,43]. Specifically, the spacing between the gain section mirrors \( z \) will be different when the thermal lensing is included than when it is not (Fig. 2) for the same initial spot size at the high reflector \( M_2 \), simply because the different focusing conditions will be required to compensate for the thermal lensing so that stable cavity oscillation is assured. This is exemplified in Fig. 6 where we depict the spot size of the intracavity laser mode \( w_L \) as it propagates through the gain medium under different combinations of lensing conditions. In the absence of any lensing (solid line), the free propagating Gaussian laser mode is focused at the center of the Ti:sapphire crystal and has a spot size of 18 \( \mu \)m – corresponding to a typical cavity mode size near the center of either stability region. When thermal lensing is included (dashed line) using the pump focusing conditions described for Fig. 5, the character (divergence and spot size) of the laser mode exiting the gain medium is clearly different from that of the non-thermal case. Consequently, a different gain mirror separation \( z \) (specifically the distance \( x \) between the exit face and the gain section mirror/lens \( f_1 \) in this case) will be required to ensure the cavity mode's wavefront in the presence of thermal lensing is matched to the plane output coupler \( M_1 \).

For comparison, Fig. 6 also shows the variation of the cavity mode size in the gain medium when Kerr lensing is included in the non-thermal (dotted) and thermal (dot-dashed) cases for the same incident focusing conditions.

![Fig. 6](image)

**Fig. 6.** The spot size \( w_L \) of a typical intracavity mode in the central portion of the gain medium under different conditions: in the absence of bulk lensing effects (solid line), with only Kerr lensing (dotted line), with only thermal lensing (dashed line), and in the presence of both thermal and Kerr lensing (dot-dashed line). In the free propagating case, the cavity mode focuses to a spot size of 18 \( \mu \)m at \( x = 10 \) mm. The pump beam focusing parameters are identical to those in Fig. 5.

and at laser power of 0.25\( P_c \). Clearly, the effect of the Kerr lensing on the cavity mode propagation is much less pronounced than that of the thermal lensing. This is to be expected in this case since the propagating cavity mode spot size of 18 \( \mu \)m, corresponding to oscillation near the center of either stability region, ensures that the intracavity irradiance, and hence nonlinear refraction, is relatively small. The Kerr lensing is significantly stronger nearer the cavity stability edges (as shown in Fig. 2) where the focal spot size of the intracavity radiation is reduced.

The effect of the pump-induced distributed thermal lens in the Ti:sapphire gain medium on the cavity stability limits is more dramatically evident in Fig. 7 where we show a contour plot as a function of \( x \) and \( z \) when the pump-induced thermal lens is included in the cavity simulation. The dashed lines indicate the resonator stability edges in the absence of thermal lensing. The grey scaling is identical to that in Fig. 2.

![Fig. 7](image)

**Fig. 7.** Contour plot of the Kerr-lens sensitivity parameter \( \delta \) as a function of \( x \) and \( z \) when the pump-induced thermal lens is included in the cavity simulation. The dashed lines indicate the resonator stability edges in the absence of thermal lensing. The grey scaling is identical to that in Fig. 2.

The distortion in the stability limits is most noticeable at \((z, x) = (102.4 \) mm, 40.0 \) mm\) where for our resonator, with \( d_1 = 80 \) cm and \( d_2 = 100 \) cm, the two inner stability edges just come in contact with each other. The two stability regions overlap at \( x \approx 40 \) mm if the thermal lens becomes stronger (due to an increased pump power, lower...
thermal conductivity, etc.) and/or if the resonator is more symmetric (e.g. \(d_1 = 85\) cm and \(d_2 = 95\) cm). This introduces a degeneracy in the possible values of \(z\) for which the resonator is stable, i.e. the laser can oscillate with two different spatial mode profiles for the same separation of the focusing mirrors in the gain section of the resonator.

Another striking result of the thermo-optic effect is that ML operation around \(x = 39\) mm in stability region I, which was the most strongly favored region for KLM operation in the absence of thermal lensing effects (see Fig. 2), has been greatly perturbed by the presence of the distributed thermal lens. Specifically, most of the areas with \(\delta < -0.6\) in the non-thermal case have disappeared, resulting in the favored regions for ML operation being more strongly confined to the inner stability edge of region I. On the other hand, the integrity of KLM operation at around \(x = 53\) mm in stability region II is not nearly as strongly perturbed by the presence of the pump-induced thermal lens. This is consistent with the experimental observation of Cerullo et al. [10] that the preferred point for high power and stable hard-apertured KLM operation is in region II at a distance of \(\sim 0.5\) mm from the inner stability edge. For reasonable fixed pump focusing conditions, operation closer to the stability edge is hindered by a reduction in the pump/laser overlap in the gain medium which would result in a reduced intracavity power and a higher oscillation threshold.

The results presented in Fig. 7 also illustrate that a deeper understanding of the KLM mechanism in Ti:sapphire and other solid-state lasers is necessary. This is clearly the case since ML operation is possible in stability region I even for symmetric resonators (i.e. \(d_1 = d_2\)) [11–13], despite that fact that this analysis indicates that the thermo-optic effect would make hard-aperture KLM difficult in this stability region. In contrast, recent numerical simulations of KLM Ti:sapphire resonators show that the gain guiding plays a more important role in the modelocking mechanism in stability region I than in region II [19]. A complete explanation will probably require a more accurate model of the resonator which includes a Brewster-cut gain medium, astigmatism compensation, and the propagation of elliptical pump and laser beams through the gain medium in the presence of gain guiding and thermal lensing.

We emphasize that the results presented in Fig. 7 are not dependent upon the propagation direction of the pump beam. In fact, if the pump beam propagates in the opposite direction (i.e. towards the high reflector \(M_2\)) our numerical simulation predicts less than a 5% change in the value of the Kerr-lens sensitivity parameter \(\delta\) and almost no change in the positions of the stability regions. Some differences are to be expected if \(aL > 4\) since the thermal lens will become more confined to one end of the laser gain medium. However, the propagation direction of the pump beam will influence the lasing efficiency since the pumping will be more efficient if the pump beam enters the gain medium at the end nearest the focal point of the cavity mode – a higher pump irradiance resulting in increased inversion before absorption effects dominate.

### 7. Thermal expansion and stress

The pump-induced heating of the Ti:sapphire gain medium causes two other effects: (i) thermal expansion of the ends of the laser rod [26,34,43], and (ii) stress in the bulk of the solid-state gain medium. These two effects are related to each other in that the expansion is limited to a small region near the end of the laser rod, which results in the accumulation of stress throughout the solid-state gain medium, since the bulk of the sapphire host crystal does not permit the whole of the laser gain medium to expand uniformly. The thermal expansion influences the propagation of the pump and laser beams in the gain medium by generating a convex surface on the ends of the gain medium so that the Ti:sapphire rod acts like a 20 mm-thick positive lens. On the other hand, the thermal stress produces an additional, but radially asymmetric, contribution to the bulk distributed thermal lensing.

The strength of the lensing caused by the thermal expansion can be estimated by comparison with flashlamp-pumped solid-state lasers where only material within a distance of approximately one rod radius from the rod end contributes to the distortion of the flatness of the rod ends [34]. In our case, the heat source is provided by 6 W from an \(\text{Ar}^+\) laser which, depending on the value of \(x\), has a spot size \(w_p\) between about 12 and 200 \(\mu\)m at the rod ends. So, by analogy, only material within \(\sim w_p\) of the rod faces would be expected to contribute to the thermal expansion. Using this estimate and the weighted parabolic approximation to the thermal lens used in Section 5, an expression for the radius of curvature \(R\) associated with the distortion of the rod ends may be found:

\[
R \approx \frac{\pi A_T \kappa w_p}{2 \alpha X P_p},
\]

where \(P_p\) is the pump laser power at the rod face and the linear expansion coefficient \(\kappa\) has a value of \(5.8 \times 10^{-6}\) K\(^{-1}\) for sapphire [34]. The minimum value of \(R\) will occur on the incident rod face when the 6 W pump beam is focused at the rod face, i.e. \(w_p = 12\) \(\mu\)m. For \(A_T \sim 1\), this gives a minimum value of \(R\) of the order of 1 m which, when compared to the 5 cm focal length mirrors in the gain section and the divergence of both the pump and laser beams in the vicinity of the laser rod, strongly suggests that thermal expansion of the gain medium is a negligible effect in Ti:sapphire laser oscillators. This effect has therefore been omitted in our analysis of the pump-induced thermal lensing, but it may become more important for solid-state gain media with lower thermal conductivities [26].
The refractive index changes due to thermal stresses may be evaluated using the photoelastic tensor [26,34]. However, since the stress is caused by a temperature gradient, the stress-induced refractive index change Δn experienced by the polarized intracavity laser radiation is expected to be radially anisotropic. This is because the tensor nature of the photoelastic effect ensures that the relative directions of the intracavity laser polarization, the stress and the crystallographic orientation of the gain medium affect the strength of the lensing. The exact details of the stress-induced thermal lensing depend upon the nature of the photoelastic tensor, specifically the relative strength of the off-diagonal and diagonal photoelastic tensor elements. Based on the analysis for YAG [34] and the photoelastic coefficient quoted for the acousto-optic interaction in sapphire [44], we estimate this thermal lensing mechanism to be about one fifth the strength of the thermo-optic effect. Consequently, the stress contribution to thermal lensing should be included in more complete models of KLM solid-state lasers which separate the tangential and sagittal planes in Brewster-cut gain media and so are equipped to handle the asymmetric lensing effects. However, the radially asymmetric nature of this effect precludes its inclusion in the radial analysis presented in this work.

8. Summary

In summary, we have demonstrated using a radial GRIN lens Gaussian beam propagation code that the pump-induced distributed thermal lens has a significant impact on the operational characteristics of KLM Ti:sapphire lasers producing ~ 100 fs pulse durations. Specifically, the thermal lensing is shown to strongly perturb ML operation in stability region I - the "High Misalignment Sensitivity" region of Ref. [10]. Furthermore, the static thermal lens both shifts and distorts the CW stability regions of the laser resonator, allowing the two stability regions to overlap as the thermal lens increases in strength and/or the resonator becomes more symmetric (i.e. d1 = d2). Both of these thermal effects have a minimal effect on ML operation in stability region II - the "Low Misalignment Sensitivity" region [10,45]. We do not expect the inclusion of a Brewster-cut gain medium to substantially alter the results presented in this paper since a similar refinement in the modeling of Ti:sapphire resonators with just the Kerr lens (compare Refs. [10] and [11] with Ref. [12]) did not alter the results significantly. Nevertheless, a clearer understanding of the role of thermal lensing effects, including stress dependent anisotropic refractive index changes, will require a more complete simulation with astigmatism compensation.

The results of our modeling of the pump-induced thermal lensing are somewhat dependent on the choice of pump beam focusing parameters. However, for the chosen pump beam parameters, which provide close to the optimum pump/laser overlap in the gain medium throughout most of the stability regions, the gross features of the results are not strongly dependent upon small variations in the exact pumping conditions. Nevertheless, we accept that pump focusing conditions can be chosen so that the pump/laser overlap is optimized for oscillation nearer the inner cavity stability edges where KLM operation is more strongly favored, or so that gain guiding and saturation effects [16–19] play a more prominent role in the KLM mechanism.

We emphasize that the results presented in this paper are based entirely on a canonical Ar+-pumped Ti:sapphire femtosecond laser system. Qualitatively similar results are expected for laser-pumped Kerr-lens modelocked lasers employing other solid-state gain media for which dn/dT is positive (e.g. forsterite, YAG, etc.) [34]. However, for similar pump powers, values of χ, crystal lengths and Stokes shifts, the influence of the thermal lens should be even stronger in these materials, since sapphire has some of the best thermal properties amongst solid-state laser materials [34]. Moreover, significantly different results are expected for materials such as LiSAF, LiCAF and YLF for which a negative dn/dT produces a negative thermal lens [46].

Finally, it should be noted that for KLM solid-state lasers generating optical pulses with a duration of ~ 10 fs [7,8], the influence of the thermal lens is expected to be less pronounced since these lasers operate at peak pulse powers equal to 2–3 times the critical power. As a result, the intracavity laser mode experiences a nonlinear Kerr lens in the short ~ 2 mm-long gain medium which, despite the effects of linear dispersion [21], is generally an order of magnitude stronger than that used in this paper. The strength of the thermal lens per unit length of the gain medium is also increased in these lasers since typically αL ~ 1, but this is more than offset by the increase in intracavity pulse power.

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References