Numerical analysis of the output power of the injection-locked cw Ti:sapphire lasers

Qinghong Zhou, Lинфang Chen, Xinye Xu *

State Key Laboratory of Precision Spectroscop and Department of Physics, East China Normal University, Shanghai 200062, China

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The optimization analysis of the output power of the injection-locked cw Ti:sapphire lasers is presented based on the fact that the injection-locked and free-running lasers almost have the same maximum output power. With the modified Ti:sapphire laser model, the dependences of the threshold and slope efficiency on the ring cavity and crystal parameters are studied through the numerical calculations, which clarify the roles of various parameters in affecting the output power. Our calculated results are in good agreement with the reported experimental data for the laser at 756 nm. Therefore our numerically calculated results could be used as a guideline for designing and optimizing such kind of the lasers.

1. Introduction

The high-power and low-noise laser sources are always chased in the fields of laser spectroscopy, nonlinear optics and laser cooling and trapping. The injection locking is one of the most efficient techniques to be applied for realizing such lasers. Up to now, the injection locking has been demonstrated in various cw laser systems, such as argon ion [1], dye [2], diode [3], and Nd:YAG [4–6]. This technique typically is accomplished by injecting a low-power master laser into a high-power slave laser. With the feedback control method, such as the Pound–Drever–Hall (PDH) method, the slave laser can be operated by following with the mode of the master laser. This is the unique technical character distinguished other laser systems, so that the stable single-frequency, high-power and low-noise lasers can be generated. Since the low noise (phase and intensity) character is the most important nature of the injection-locked lasers, the related theoretical works have been carried out. For instance, authors in Ref. [7] have calculated the intensity and frequency noises by the transfer function; authors in the Ref. [8] have proposed a theoretical mode based on the linearized input–output method to calculate the intensity noise spectrum for a general injection locking. For an injection-locked Ti:sapphire laser the intensity noise has been also analyzed in reference [9]. On the other hand, recently several experiments on demonstrating the injection-locked cw Ti:sapphire lasers have been reported [10–12] with the advantages of high efficiency, wide tuning range and high stability. One of significant applications is that a watt-level 378 nm laser has been generated by the 5 W 756 nm injection-locked Ti:sapphire output in Ref. [12]. Comparing with other traditional method [13,14] to realize the single mode operation in Ti:sapphire laser, this method is more effective but difficult to do the system construction in the experiment. Since in the experiment the ring cavity is hard to regulate and optimize per se, and moreover the ring cavity needs to be mode matched with the master laser beam as well as the pump beam simultaneously, a detailed optimization to guide the experiment is needed. Some studies for optimizing such as the mode matching parameters, the material figure of merit (FOM), the rod length and the absorption coefficient for pump beam to maximize the output power of such lasers have been also reported [15–18]. In some references the optimizations are discussed for a general situation. The references [15–18] discussed the influence of the beam sizes and the mode matching parameters in general. Actually, the output power of the injection-locked Ti:sapphire laser is mainly determined by the laser and pump beam sizes in the crystal, but these beam sizes are mainly determined by the ring cavity structure. Moreover, this cavity is difficult to be adjusted, because these beam sizes are very sensitive to the cavity structure. To our knowledge, the optimization for the output power with the ring cavity structure has not been reported. In addition, the Ti:sapphire crystal parameters also affect the output power significantly, which was recently discussed in the Refs. [16] and [18] by using the analytic methods. However detailed numerical analyses for optimizing such laser performance with these parameters have not been done yet. Fortunately, the output power of the injection-locked laser could be optimized by analyzing the output power of the slave laser based on the fact that the injection-locked and free-running lasers almost have the same maximum output power. Here we analyze the dependences of the output power on the cavity and Ti:sapphire crystal parameters to give a clear guideline for optimization.

In this paper, based on the characteristics of injection-locked lasers, we focus on discussing how to optimize the output power of
the slave laser by analyzing the influence of various parameters (including cavity structure and crystal parameters) on the threshold and slope efficiency using a modified theoretical model [16]. With the numerical method, the detailed analyses are applied to a Ti:sapphire laser at 759 nm pumped with a 532 nm laser. The numerically calculated results are compared with the reported experimental results for a 3-W Ti:sapphire laser at 756 nm pumped with a 11.5-W laser at 514 nm [11]. Finally a good agreement is found, and the further improvement is suggested.

2. Theory and numerical modeling

The theory of the laser injection locking has been presented in several publications [10,19]. Here we give a brief sketch. Using the subscript m (s) to represent the master laser (the free-running slave laser), we assume that a ring laser is operating at its cavity resonance frequency $\omega_m$ with the output power of $P_m$, while another laser with the frequency of $\omega_m$ and the power of $P_m$ is injected into the ring cavity through the output coupler as shown in Fig. 1. As the frequency of the injected laser $\omega_m$ is tuned close to $\omega_m$, the amplitude and the frequency of the master laser field begins to saturate the gain, and thus the injected laser at $\omega_m$ takes over the free-running laser at $\omega_m$. However, with $\omega_m$ tuned further inside the locking range, the laser power at $\omega_m$ would not increase any more.

Here, we describe the output power of the injection-locked lasers based on the equation for the time-varying amplitude of the cavity electric field outside the output coupler given by Siegman [19] as

$$\frac{dE(t)}{dt} + \frac{\gamma_m - \gamma_p}{\gamma_p} E(t) = \gamma_p E_m(t) \cos[\phi(t) - \phi_m(t)],$$

where $\gamma_p$ refers to the growth rate of the cavity field that is due to the saturated gain of the slave laser, $\gamma_p$ is the total cavity loss rate and $\gamma_m$ is the external decay rate due to output coupling. Under the steady-state injection-locked and $E_m/E_c < 1$ conditions, the first order approximate solution for the output amplitude versus the master laser frequency could be obtained as

$$E^2(\omega_m) \approx E_t^2 \left[ 1 + \frac{2r}{(r-1)} \frac{\gamma_p E_m \cos \phi(\omega_m)}{\gamma_p E_t} \right],$$

where $r = \gamma_m/\gamma_p$ is the ratio of the unsaturated laser gain to the losses in the cavity. Eq. (2) shows that the injection-locked and free-running lasers almost have the same maximum output power with the condition of $E_m/E_c < 1$. And in a general injection-locked laser system, the master laser power is indeed sufficiently smaller than the output power of the slave laser, for this reason the slave laser determines how much output power the injection-locked laser can provide. Thus the optimization of the injection-locked Ti:sapphire laser output power could be implemented by analyzing the output power of the slave laser based on a Ti:sapphire laser model.

The theoretical model for a longitudinally pumped cw Ti:sapphire laser in a standing-wave cavity had been presented in Ref. [16]. The similar theoretical model in a ring cavity could be given by

$$P_p = \frac{(T + \frac{\alpha L}{FOM} + \eta) h \pi}{4 \alpha \tau_p \alpha_p} \int_0^\infty e^{-\omega^2} Q(z) dz,$$

where $Q(z) = \frac{1}{w_c(z)w_c(z)w_p(z)w_p(z)} \int_0^\infty \int_0^\infty e^{-\omega^2} dx dy$, $A_x = \frac{2}{w_p(z)} \left( \frac{w_p(z)}{w_c(z)} \right)^2$, $A_y = \frac{2}{w_p(z)} \left( \frac{w_p(z)}{w_c(z)} \right)^2$, $D_x = \frac{2}{w_p(z)}$, $D_y = \frac{2}{w_p(z)}$, $B = \frac{2 s P_e}{m w_c(z)w_c(z)}$, $w_p(z) = w_{p0} \left[ 1 + \frac{(z-z_0)^2}{\lambda w_{p0}^2} \right]$, $w_c(z) = w_{c0} \left[ 1 + \frac{(z-z_0)^2}{\lambda w_{c0}^2} \right]$, $w_{p0} = \frac{w_{p0}^2}{w_{p0}^2}$, $\theta_0 = \frac{\pi}{\lambda w_{p0}^2}$. The subscript (p) refers to the parameters of the cavity oscillation laser (pump laser). $P$ is the laser power, $\lambda$ is the laser frequency, $\lambda$ is the laser wavelength, $h$ is Planck constant, $\alpha$ is the emission cross section, $\alpha$ is the lifetime of fluorescence, $s$ is the saturation parameter, $n$ is the refractive index of Ti:sapphire crystal, $T$ is the crystal rod length, $\alpha$ is the absorption coefficient of Ti:sapphire crystal, $FOM = \gamma_m/\gamma_p$, $\sigma$ is the material figure of merit of the crystal rod, $T$ is the output coupler transmission and $\eta$ is the insertion loss of intracavity optics. $w_{p0}(z)$, $w_{p0}(z)$, $w_{c0}(z)$ and $w_{c0}(z)$ are the pump and cavity beam radii variations along the propagation $z$ in the tangential and sagittal planes, respectively. $w_{p0}$, $w_{p0}$, $w_{c0}$ and $w_{c0}$ are the corresponding pump and cavity beam waists in the crystal centre. Eq. (3) indicates the implicit relationship between $P_p$ and $P_r$ contained within the parameter $Q(z)$. It could not be written to an explicit expression in terms of output power $P_{out}$ (obtained by multiplying $T$ and $P_r$) and $P_p$. However, in the limit of $P_r \rightarrow 0$, from Eq. (3) the threshold could be given as

$$P_{th} = \frac{(T + \frac{\alpha L}{FOM} + \eta) h \pi}{2 \alpha \tau_p \alpha_p} \int_0^\infty \frac{1}{\sqrt{w_c(z)w_c(z)w_p(z)w_p(z)}} \int_0^\infty \frac{1}{\sqrt{w_c(z)w_c(z)w_p(z)w_p(z)}} \int_0^\infty \frac{1}{\sqrt{w_c(z)w_c(z)w_p(z)w_p(z)}} e^{-\alpha r^2} dz.$$
on 759 nm with the 532 nm pump laser. The distance between the crystal end face and the closer curved mirror is d and the longer distance between the two curved mirrors is D. These two parameters combining with the curvature radius R are the main factors of the ring cavity structure to determine the laser beam size in the crystal. In the experiment, the curved mirrors are set at an angle to compensate the astigmatism, and the optical diode is used for unidirectional lasing. The piezoelectric transducer (PZT) mounted on the flat mirror, the fast photodiode and the feedback circuit constitute the PDH system to realize the injection locking by controlling the cavity length and master laser current. In the following numerical analysis, we analyze the influence of various parameters on the output power. However, the influence of the different values of FOM has been discussed [16], which suggests that the increasing in FOM (decreasing in α) is accompanied by the increasing in the output power, but this tendency diminishes for FOM > 150. Meanwhile, considering the manufacturing limitation this value is usually selected around 150. Another parameter with the clear influence on the output power is the intracavity loss η, which should be as small as possible according Eq. (3). Now we mainly discuss the influence of the parameters D, R, d, L, αp, and T on the threshold and slope efficiency in order to optimize the output power. We firstly analyze the dependence of the threshold and slope efficiency on the pump beam waist for a given cavity mode waist in the crystal centre. Then with the optimal ratio of the pump to laser beam waists for each point in the stability zone, we present the dependences of the threshold and slope efficiency on the cavity parameters (D, R, and d) to give a guideline for designing an optimal cavity. Additionally, we discuss the influences of the crystal parameters (L and αp) and output coupler transmission T on the output power in the same way. At last, we make the comparison between our numerical calculation results and the reported experimental data of a 3 W cw Ti: sapphire laser at 756 nm to examine our theoretical model.

3.1. Optimum pump beam for a given cavity mode waist

For a given cavity mode waist, the pump beam waist is an important factor in determining the threshold and slope efficiency. Fig. 2(a) and (b) shows the results with the parameters D = 0.6 m, d = 0.053 m, R = 0.1 m, T = 10%, η = 0.04, L = 1 cm, αp = 2.5 cm⁻¹, FOM = 200, and the saturation intensity calculated to be 2.78 × 10⁵ W/cm² from Ref. [18]. These plots indicate that there is an optimum pump waist for achieving the lowest threshold with the greatest slope efficiency, and the corresponding values of wpxo/Wcxo and wpxo/Wcyo are about 0.5 and 0.3, respectively. Additionally, the degree of the mode matching between the cavity and pump modes is given by Fig. 2(c) with the same parameters as above by using the overlapping efficiency [20]. The result shows that the optimum ratios for achieving the highest mode matching efficiency are also 0.5 and 0.3, as well as the situation of obtaining the highest output power.

3.2. The threshold and slope efficient versus the cavity parameters

In the experiment, the ring cavity structure is not quantified precisely, but adjusted usually according experience. This may be leading an imperfect optimization. Therefore a theoretical analysis about the cavity structure is necessary. The above results clearly suggest that there is an optimum pump waist for achieving the maximum output power with the given cavity mode waist. And in this way, we discuss the influence of the cavity parameters on the output power by looking for the optimum pump waist of each point in the stability region. By this method, the influence of the cavity structure on the output power can be extracted. The contour plots of the threshold and slope efficiency versus parameters D, d and R within the stability region are given as Fig. 3. The plots in Fig. 3(a1), (b1), (a2) and (b2) illustrate that the values of d (or R) corresponding to the center of the stability region bring the slope efficiency and threshold maximum for a fixed R (or d) and D, and both of them decrease gradually with the parameter d (or R) away from the center of the stability region, whereas the parameters d and R have a much more influence on the threshold than slope efficiency, which could be clearly seen from the plots shown in Fig. 3(a) and (b). And from Fig. 3(a1), (b1), (a2) and (b2) one can also see that the decreasing in D causes a gradually increasing in the threshold, slope efficiency and stability zone area. The laser beam radius in the crystal in the center of the stability region is larger than that on the edge of the stability region, which causes a higher gain on the edge of the stability region. And the larger D values also have a similar effect. Therefore, these plots suggest that a larger value of D and the corresponding parameters R, d in the off-centre stability region should be chosen to achieve a higher output power, which could be a good guideline for designing the cavity configuration.

3.3. The threshold and slope efficiency versus the output coupler transmission, absorption coefficient of the pump beam and rod length

The transmission of the output coupler, the absorption coefficient of the pump beam and the rod length also influence the output power significantly. Some analytic results have been reported [16]; here we give the numerical analysis results, which are more precise and comprehensive. The contour plots of the threshold and slope efficiency versus different values of αp and T with L = 1 cm, and T with αp = 2.5 cm⁻¹ are shown in Fig. 4(a1), (b1), (a2) and (b2), respectively. These plots demonstrate that there is an optimum value of αp (L) to achieve a lowest threshold with highest slope efficiency for the given L (αp) and T, and the optimal αp (L) is nearly about 3.5 cm⁻¹ (0.8 cm) for L = 1 cm (αp = 2.5 cm⁻¹). The results could be clearly

Fig. 2. Contour plots of the threshold (a), slope efficiency (b) and overlapping efficiency (c) versus the ratio of the pump to cavity mode waist in the sagittal and tangential planes, respectively, with the parameters R = 0.1 m, D = 0.6 m, d = 0.053 m, L = 1 cm, T = 10%, αp = 2.5 cm⁻¹, FOM = 200 and η = 0.04.
seen from the black dashed curve shown in Fig. 4(a) and (b) that the product of \( \alpha_p \) and \( L \) is almost constant for achieving the optimal output power, and a decrease in \( L \) is accompanied by an increase in the slope efficiency, but a decrease in the threshold. According to Eq. (3), the product of \( \alpha_p L \) is related to the absorption of laser beam by FOM in the crystal, which can be seen as a loss in the cavity. On the other hand, it also determines how much pump power can be absorbed. Therefore there certainly is an optimal value of this product. The output power could be improved by shortening the rod length, but at a larger expense of \( \alpha_p \). In practice, the rod length usually used is about 1 cm [10–12] for achieving a higher output power due to the technique limitation of \( \alpha_p \). Fig. 4(a), (b), (c) and (d) also illustrates that the increasing in \( T \) causes the sharp increasing in the threshold and slope efficiency, which suggests that there is an optimum \( T \) to achieve a highest output power for the given \( \alpha_p \) and \( L \). Similarly the output coupler transmission \( T \) can cause the cavity loss; on the other

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**Fig. 3.** Contour plots of the threshold (a) and slope efficiency (b) versus different values of \( D, R \) and \( d \) with other parameters as Fig. 2 and the optimal \( w_{p\alpha}/w_{c\alpha} \) and \( w_{p\alpha}/w_{c\alpha} \) for each point, where the yellow curves represent the boundary of the stability region. The threshold (a1) and slope efficiency (b1) versus \( D \) and \( d \) with \( R = 0.1 \) m; the threshold (a2) and slope efficiency (b2) versus \( D \) and \( R \) with \( d = 0.053 \) m; and the threshold (a3) and slope efficiency (b3) versus \( R \) and \( d \) with \( D = 0.6 \) m.

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**Fig. 4.** Contour plots of the threshold (a) and slope efficiency (b) versus different \( T \) and \( \alpha_p \), \( T \) and \( L \), and \( \alpha_p \) with other parameters as Fig. 2 and the optimal \( w_{p\alpha}/w_{c\alpha} \) and \( w_{p\alpha}/w_{c\alpha} \) for each point. The threshold (a1) and slope efficiency (b1) versus \( \alpha_p \) and \( T \) with \( L = 1 \) cm; the threshold (a2) and slope efficiency (b2) versus \( L \) and \( T \) with \( \alpha_p = 2.5 \) cm\(^{-1}\); the threshold (a3) and slope efficiency (b3) versus \( \alpha_p \) and \( L \) with \( T = 10\% \) with the black dashed curves represented the optimal relationship between \( \alpha_p \) and the optimal \( L \).
4. Comparison with experimental results

To examine our analysis, we make the comparison between our numerical calculation result (solid line) and the reported experimental data (black dots) of a 3 W cw Ti:sapphire laser at 756 nm pumped with a 11.5-W laser at 514 nm [11], which is shown in Fig. 5. The parameters adopted in the calculation are: $R = 0.1 \text{ m}$, $L = 1 \text{ cm}$, $D = 0.58 \text{ m}$, $d = 0.057 \text{ m}$, $\alpha_P = 2.8 \text{ cm}^{-1}$, $\text{FOM} = 160$ and $T = 10\%$. It can be found from Fig. 5 that the calculated result is well agreed with the reported result. And by optimizing parameters, the output power and slope efficiency could be increased to 3.7 W and 42.6%, respectively, which could be seen from the dashed line with the parameters $R = 0.1 \text{ m}$, $L = 1 \text{ cm}$, $D = 0.6 \text{ m}$, $d = 0.0575 \text{ m}$, $\alpha_P = 3.8 \text{ cm}^{-1}$, $T = 15\%$, $\eta = 0.06$, $w_{\text{RRO}}/w_{\text{CAB}} = 0.55$, $w_{\text{RRO}}/w_{\text{CYO}} = 0.35$ and $\text{FOM} = 160$ and others as above. The result suggests that there is a potential improvement over the reported experimental result, especially for adjusting the cavity structure parameters $D$ and $d$, the absorption coefficient $\alpha_P$ and the transmission of the output coupler $T$.

5. Conclusion

In conclusion, we have employed the numerically calculating method to analyze the dependences of the threshold and slope efficiency on various parameters including the cavity structure and crystal parameters for optimizing the output power of a cw injection-locked Ti:sapphire laser. The results clearly show that the optimum ratios of $w_{\text{RRO}}/w_{\text{CAB}}$ and $w_{\text{RRO}}/w_{\text{CYO}}$ for achieving the highest mode matching efficiency are the same as that of the highest output power. And we have found that the cavity parameters have more sensitivity to the threshold than the slope efficiency, and a larger value of $D$ and the corresponding $R$, $d$ in the off-centre stability region should be chosen to achieve a higher output power, which could be a good guideline for designing the cavity configuration. Additionally, the product of $\alpha_P$ and $L$ is almost constant for achieving the optimal output power, and the power could be improved by shortening the rod length, but at a larger expense of $\alpha_P$. And there is an optimum $T$ to achieve the highest output power for the given $\alpha_P$ and $L$. At last, a good agreement between the calculated result and the reported experimental data of a 3 W laser at 756 nm validates our analysis, and we further predict that the output power of the demonstrated laser could be increased to 3.7 W by optimizing parameters. However, the influence of the thermal lens cannot be ignored with the high pump power in practice, and the dynamic and noise property in the high power region also needs to be addressed. Therefore, we will further improve our analysis by taking into account these effects in the future.

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References