A Faraday isolator (FI) with depolarization compensation using a counterrotation scheme has been realized in experiment for the first time, to the best of our knowledge. It is based on terbium scandium aluminum garnet crystals with negative optical anisotropy parameters. An order of magnitude advantage over the traditional FI scheme is achieved in this case. An isolation ratio of 35.7 dB at a radiation power of 1440 W has been obtained. According to the numerical estimates, an isolation ratio of 30 dB can be provided up to a power of 5.5 kW.

The traditional FI scheme consists of a Faraday rotator and a half-wave plate which are placed between two polarizers with different orientations. The rotation of the polarization plane is generated by the Faraday rotator, and the depolarization is compensated by the half-wave plate.

The photoelastic component of depolarization arising under the action of the photoelastic effect can be calculated from [16],[3]:

\[ \gamma = \gamma_{ph} + \gamma_{VH}, \]  

where \( \gamma \) is the total angular rotation of the polarization plane in FI, \( \gamma_{ph} \) is the photoelastic component, and \( \gamma_{VH} \) is the Verdet component.

As the angle of rotation of the polarization plane in FI is \( \theta_F = \pi/4 \) rad, the \( \gamma_{VH} \) component can be calculated from [16]

\[ \gamma_{VH} = A_3 \rho_V^2, \]

\[ \rho_V = \frac{\alpha}{16\kappa} \left( P_0 + P_0^* \right) \left( \frac{1}{V} \frac{dV}{dT} + \alpha_T \right), \]

\[ P_0^* = -\delta H \cdot T_0 \frac{4\pi \kappa r_b^2}{\alpha R_0^3}, \]  

where \( A_3 \) is the numerical coefficient dependent on the beam shape (for a Gaussian beam \( A_3 = 0.00104 \)), \( \alpha \) is the coefficient of medium absorption, \( \kappa \) is the thermal conductivity, \( P_0 \) is the laser radiation power, \( V \) is the Verdet constant of the system, \( T_0 \) is the temperature, \( \alpha_T \) is the linear expansion coefficient, and \( r_b \) is the effective beam radius. The dependence of the magnetic field on the transverse coordinate is taken in the form \( H(r) = H_0 (1 + \delta H \cdot r^2/R_0^2) \), where \( R_0 \) is the MOE radius.

The photoelastic component of depolarization \( \gamma_{ph} \) depends on the chosen FI scheme and was earlier [3] calculated analytically for media with weak linear birefringence \( \delta_I \) corresponding to the condition

\[ |\delta_I| \xi < 1. \]  

The traditional FI scheme consists of a Faraday rotator and a half-wave plate which are placed between two polarizers with...
parallel axes. For a Faraday rotator with a single MOE in a magnetic field \(\theta_F = \pi/4\), Fig. 1(a), \(\gamma_{ph}\) may be calculated by the formula

\[
\gamma_{ph}(|\delta\xi| \ll 1) = \frac{A_1}{\pi^2} p^2, \quad (4)
\]

\[
p = \frac{\alpha P_0 L Q}{\kappa \lambda} \cdot \quad (5)
\]

Here, \(A_1\) is the numerical coefficient dependent on beam shape (for a Gaussian beam \(A_1 = 0.137\)), \(p\) is normalized heat release power, \(L\) is the MOE length, \(Q\) is the thermo-optical constant of the medium, and \(\lambda\) is the radiation wavelength. The formula is written for the [001] crystal orientation and an optimal angle between one of the crystal axes and incident radiation polarization \(\theta = \pi/8\).

For media in magnetic field with \(p \ll 1\) but \(|\xi| \gg 1\), expression (4) is supplemented by one more term proportional to \((\rho \xi)^4\) [13]:

\[
\gamma_{ph} = \frac{A_1}{\pi^2} p^2 + \frac{3(\pi - 2)^2 A_2}{32 \pi^4} p^4 \xi^4, \quad (6)
\]

where \(A_2\) is the numerical constant \((A_2 = 0.042\) for the Gaussian beam). The depolarization in such media is proportional to the fourth power of the radiation power, rather than to its square and greatly exceeds thermally induced depolarization in the absence of magnetic field.

For compensating for \(\gamma_{ph}\) more complicated Faraday rotator schemes are needed. The main idea is to use two optical elements in which beam distortions compensate for each other—they are accumulated in one MOE and are compensated for in the other one. The most frequently employed scheme is depicted in Fig. 1(b). Two MOEs are used instead of one, each rotating the polarization by \(\theta_{F,1,2} = \pi/8\) rad, with a reciprocal polarization rotator by \(\theta_{rot} = 3\pi/8\) rad placed in between [3]. The photoelastic part of thermally induced depolarization may be calculated as

\[
\gamma_{ph} = 3 A_1\left(\pi - 2 \frac{2}{\sqrt{2}}\right)^2 \frac{p^2}{\pi^2} \left(1 + \frac{2}{3} \frac{\delta^2}{\xi^2} + \frac{\delta^4}{\xi^4}\right), \quad (7)
\]

where \(p\) is the normalized heat release power in each MOE. The result of numerical integration of the Jones matrices for TSAG is presented in Fig. 2 (dotted curve).

The main drawback of the method is the need to place an additional element (reciprocal rotator) in the center of the magnetic system, where the magnetic field is the strongest, as a result of which the MOE are displaced to the region of a weaker field. In practice, the length of each displaced MOE providing \(\theta_{F,1,2} = \pi/8\) is actually equal to the MOE length in a standard scheme [17,18].

For active laser elements (AE), the counterrotation scheme consists of two cylindrically shaped coaxial AEs made of the same cubic crystal, have the same length and orientations defined by the Euler angles \((\rho, \xi, \chi)\) and \((\rho_2, \xi_2, \chi_2)\) = \((\pi + \rho, \pi - \xi, -\chi)\), respectively [19]. In this case, under certain conditions on the angles \(\rho, \xi\) and the \(\xi\) parameter by choosing the optimal angle \(\chi\) compensation of the part of photoelastic depolarization proportional to squared normalized heat release power can be achieved [19].

Let us consider this scheme for MOEs in the [001] orientation \((\rho = \xi = 0)\). An important advantage of this scheme is the absence of a reciprocal rotator, so both crystals may be placed in the center of the magnetic system. The total length of the two crystals is equal to that of a single crystal in a traditional scheme and each crystal has a half-normalized power.

Let us find analytical expressions for depolarization and optimal angles of rotation \(\chi\) in the scheme of compensation with counterrotation. For two MOEs arranged successively with circular birefringence \(\delta_{1c} = 2\theta_{F,1}\) and \(\delta_{2c} = 2\theta_{F,2}\), the local depolarization \(\Gamma\) in the weak birefringence approximation \((\delta_i \ll 1)\) is written in the form

\[
\Gamma = \left(\frac{\sin(\delta_{1c}/2)}{\delta_{1c}}\right) \left(\frac{\sin(\delta_{2c}/2)}{\delta_{2c}}\right) \left(\frac{\sin(\delta_{1c}/2)}{\delta_{1c}}\right) \left(\frac{\sin(\delta_{2c}/2)}{\delta_{2c}}\right) \left(2 \Psi_{1,2} \frac{\delta_{1c}}{\delta_{2c}} - \frac{\delta_{1c}}{\delta_{2c}}\right) + O(\delta_i^4), \quad (8)
\]

where \(\Psi_{1,2}\) are the tilt angles of eigenpolarizations to the polarization plane of input radiation. The integral value of depolarization \(\gamma_{ph}\) for a beam with normalized intensity profile \(F(r/r_h) \int_0^{r_h} F(r)dr = 1/(2\pi)\) having half-width \(r_h\) is calculated as

\[
\gamma_{ph} = \frac{\alpha P_0 L}{\kappa \lambda} \int_0^{r_h} \frac{F(r)dr}{r} = \frac{\alpha P_0 L}{\kappa \lambda} \frac{1}{2\pi r_h} \int_0^{r_h} F(r)dr = \frac{\alpha P_0 L}{\kappa \lambda} \frac{1}{2\pi r_h} \frac{1}{2\pi} = \frac{\alpha P_0 L}{8\kappa \lambda}.
\]

**Fig. 2.** Depolarization versus radiation power. Squares, measurements of traditional Fl with double-length crystal; circles, scheme with counterrotation with normalized heat release power \(p_0\), solid curves are their simulations; dotted-dashed curve, the result of analytical calculations for a counterrotation scheme; dashed curve, simulations of the scheme with counterrotation with half-length crystals; dotted curve, scheme with compensation with reciprocal rotation.

**Fig. 1.** Faraday rotator schemes: (a) without compensation \((\theta_F = \pi/4)\), (b) traditional compensation scheme \((\theta_{F,1} = \theta_{F,2} = \pi/8; \theta_{rot} = 3\pi/8)\), (c) compensation in a scheme with counterrotation \((\theta_{F,1} = \theta_{F,2} = \pi/8)\). 1, magnetic system; 2, MOE; 3, radiation propagation direction; 4, reciprocal rotator.
Using the notation introduced in [15],

\[\delta_{ij} \sin 2\Psi_j = p_i a_j, \quad \delta_{ij} \cos 2\Psi_j = p_i b_j, \quad (10)\]

we can write the coefficients \(a\) and \(b\) for the elements manufactured from a single crystal with the [001] orientation in the form

\[a_j = \frac{b}{2} [ (\xi_j + 1) \sin (2\varphi) + (1 - \xi_j) \sin (4\theta_j - 2\varphi) ], \quad b_j = \frac{b}{2} [ (\xi_j + 1) \cos (2\varphi) + (1 - \xi_j) \cos (4\theta_j - 2\varphi) ], \quad (11)\]

where \(b\) is a function of \(F_\lambda\).

In the case of the same value of Faraday rotation \((\delta_1 = \delta_2 = \delta)\), the same material and length of each element \((p_1 = p_2 = p\) and \(\xi_1 = \xi_2 = \xi)\), but different angles \(\theta\) \((\theta_1 \neq \theta_2)\), the substitution of (10) into (8) yields

\[
\Gamma = p^2 \frac{\sin^2(\delta/2) - \sin^2(\delta_1/2) - \sin^2(\delta_2/2)}{2} - a_1 \cos(\delta/2) - b_1 \sin(\delta/2)^2 + O(\delta^4). \quad (12)
\]

Let us find the conditions at which the first term in expression (12) will be equal to zero. By substituting (11) into (12) and equating to zero the expressions at \(\sin(2\varphi)\) and \(\cos(2\varphi)\), one can find the conditions for the angles \(\theta_1\) and \(\theta_2\):

\[\theta_{1,2} = \frac{\delta}{2} \pm x + \pi k/2, \quad k = 0, \pm 1, \pm 2, \ldots, \quad \cos(4\chi + \delta/2) = \frac{-\xi + 1}{\xi - 1} \cos(\delta/2). \quad (13)\]

For an arbitrary value of \(\delta\), the last condition in (13) may be fulfilled only at \(\xi \leq 0\). For a given \(\delta\), the magnitude of \(\xi\) must meet the condition \(\xi \leq \min(\tan^2(\delta/4); \cot^2(\delta/4))\) or \(\xi \geq \max(\tan^2(\delta/4); \cot^2(\delta/4))\).

For an FI, the value of circular birefringence in each element will be \(\delta_\chi = \pi/4\) (with the total angle of rotation of the polarization plane being \(\theta_{F1} + \theta_{F2} = \pi/4\)). In this case, compensation is possible at \(\xi \leq 0.039\) or \(\xi \geq 25.27\); therefore, compensation of the depolarization part proportional to the squared power for widely used TGG crystals \((\xi = 2.25)\) is impossible. For TSAG with \(\xi = -101\), the optimal angle for compensation is \(\pi \approx -\delta/4 = -\pi/16\).

Thus, using MOEs made of a single crystal with [001] orientation and optical anisotropy parameter in the range specified above, by rotating the elements it is possible to reduce thermally induced depolarization as a result of compensation of the term proportional to the squared power. Let us find the term of the next order of smallness in the expansion of thermally induced depolarization with respect to the small parameter of linear birefringence. Assume that all the elements are identical; their angles of rotation meet those in (13), and the value of circular birefringence \(\delta_\chi = \pi/4\); then

\[
\gamma_{ph} = \frac{4A_2}{\pi^4} \rho^4 \left[ \pi^2 (\pi - \sqrt{2}-1) + 32(\sqrt{2}-1)\xi + \pi^2 (\pi - \sqrt{2}-1)\xi^2 - (\sqrt{2}-1)(\xi + 1)\sqrt{\xi^2 - 2(4\sqrt{2}+7)\xi + 1} \right] + O(\rho^6). \quad (14)
\]

For TSAG crystals \((\xi \gg 1)\), this expression may be calculated approximately as

\[
\gamma_{ph} \approx \frac{4A_2}{\pi^4} \rho^4 \xi^2 \left[ \pi^2 (\pi - \sqrt{2}-1) - (\sqrt{2}-1) \right]. \quad (15)
\]

Numerical integration of the Jones matrices also shows that, using MOEs made of TSAG crystals in a system with counter-rotation [Fig. 1(c)], thermally induced depolarization may be reduced substantially. According to the calculations, optimal angles \(\theta_{1,2}\) for the first and second MOE in a wide range of normalized heat release power are \(\theta_{1,2} = \pi/8 \pm \chi\), where \(\chi \approx 0.218\). Angle \(\chi\) is a little larger than the value calculated analytically. The difference is explained by the fact that the depolarization is minimal at a certain intermediate value ensuring balance between the terms of the second and fourth orders, rather than at a zero quadratic term.

For experimental verification, samples of TSAG crystals in the form of short cylinders with a diameter of 10 mm and a length of 9 mm were used. The optical surfaces of the crystals had a 1070 nm antireflective coating. The crystals had [001] orientation—the crystallographic axis was parallel to the axis of the cylinder.

Depolarization was measured in a conventional scheme with crossed polarizers. The crystals (each with an angle of rotation of \(\theta_{F} = \pi/8\)) fixed in a water-cooled copper sleeve for stabilizing the angle of rotation of the polarization plane were placed inside the magnetic system. The specially designed magnetic system had a maximum field strength of 25 kOe [17]; the centers of the crystals were displaced from the maximum of the magnetic system field by \(\sim 11\) mm. Note that the field strength of the magnetic system was sufficient for rotating the plane of radiation polarization by \(\pi/4\) by means of a single crystal 9 mm long in the field maximum. The Faraday rotator was placed into the beam of a ytterbium fiber laser \((\lambda = 1070\) nm, \(P_0 \leq 1500\) W) after a polarizer. The polarizer was a calcite wedge separating radiation with vertical and horizontal polarizations at a small angle, providing divergence of the polarized components at the FI length smaller than the beam radius. Further, the beam radiation with the principal horizontal polarization was attenuated by about \(10^3\) times due to reflection at a pair of silica wedges, passed through the polarization analyzer (Glan prism), and was recorded by the CCD camera.

The total intensity was calculated by integrating the local intensities over the recorded frame. The depolarized component intensity \(I_d\) was measured at the angle between the polarizer axes and the analyzer \(\pi/2 + \pi/4\) corresponding to the angle of rotation of the polarization plane in the FI, and the polarized intensity component \(I_0\) at the angle \(+\pi/4\). The radiation depolarization \(\gamma\) was calculated by the formula

\[
\gamma = \frac{I_d}{I_0 + I_d}. \quad (16)
\]

The thermo-optical characteristics of the TSAG crystals produced by the OXIDE company were investigated in [12], where it was shown that for a single TSAG crystal 9 mm long the normalized heat release power \(p_0 = P_0/96000\). The FI
created on a single crystal of this type provided the isolation ratio of 35.4 dB at a radiation power of 1470 W [13].

The power dependence of radiation depolarization for two consecutive TSAG crystals was measured for two cases. In the first case, both crystals were rotated by angles \( \theta_1 = \theta_2 = \pi/8 \), which, in respect to \( \gamma_{ph} \), corresponds to a single crystal of double length turned by an optimal angle ensuring minimum depolarization. This corresponds to the traditional FI scheme with a crystal of double length with \( p = 2p_0 \) (Fig. 2, squares). In the second case, the crystals were rotated around the \( z \) axis relative to the position \( \theta = \pi/8 \) by the same angle in different directions until reaching a minimum of thermally induced depolarization. This corresponds to compensation with counterrotation with \( p = p_0 \) (Fig. 2, circles). Figure 2 also shows the simulated curves for thermally induced depolarization for these two cases (solid curves), for the traditional FI scheme with compensation using a reciprocal rotator (dotted curve), as well as for the case when crystals in the scheme with counterrotation are placed in the magnetic field maximum (dashed curve). In the latter scheme, the MOE in the traditional FI is cut into two parts which are rotated by the needed angles; the length of each element may be reduced by two times and \( p = p_0/2 \). This is an important advantage of the scheme with counterrotation over the traditional scheme with compensation.

The noticeable difference between the experiment and theory in the region of powers less than 1000 W is attributed to the relatively high level of cold depolarization in the samples, caused by the relatively poor optical quality of the crystals. At a power higher than 1000 W, good agreement between the experimental data and the simulated dependences is observed. The scheme with counterrotation allows reducing the level of thermally induced depolarization approximately 10 times, compared to the traditional scheme with double crystal length and reaching the isolation ratio of 35.7 dB at 1440 W, which is better than in the case of a single crystal in the same magnetic field [13]. With the use of two crystals of half-length, the power may be increased by a factor of two, retaining the value of the isolation ratio; according to the simulations, an isolation ratio over 30 dB may be attained up to a power of 5.5 kW (dashed curve in Fig. 2). The calculations show that in this case, at a power higher than 4 kW, the compensation scheme with counterrotation is advantageous to the scheme with a reciprocal rotator.

Note that the depolarization component \( \gamma_{VH} \) is the same in all the considered schemes because of the same nonreciprocal total angle of rotation of the polarization plane, and its contribution is taken into consideration in all simulations.

The scheme of compensating for thermally induced depolarization with counterrotation in a magnetic field using TSAG crystals with negative optical anisotropy parameter has been implemented for the first time, to the best of our knowledge. The analytical expressions describing thermally induced depolarization in this scheme have been obtained. Approximately an order of magnitude advantage over the traditional FI scheme has been attained in depolarization \( 2.7 \cdot 10^{-4} \) at a radiation power of 1440 W. Unlike the traditional scheme of compensation with a reciprocal rotator, the scheme with counterrotation allows placing the MOE in the center of the magnetic system, where the field is the strongest due to the total length of the elements being the same as in the traditional scheme. Unfortunately, absence of a more powerful laser radiation source did not allow us to obtain even more significant results at a higher power. However, according to the estimates, with the use of TSAG in the scheme with counterrotation, the isolation ratio of 30 dB at a power up to 5.5 kW may be attained; at a power exceeding 4 kW, this scheme becomes advantageous to the traditional compensation scheme.

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